

On the relation between local and geometric Lagrangians for higher spins

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Abstract. Equations of motion for free higher-spin gauge fields of any symmetry can be formulated in terms of linearised curvatures. On the other hand, gauge invariance alone does not fix the form of the corresponding actions which, in addition, either contain higher derivatives or involve inverse powers of the d'Alembertian operator, thus introducing possible subtleties in degrees of freedom count. We suggest a path to avoid ambiguities, starting from local, unconstrained Lagrangians previously proposed, and integrating out the auxiliary fields from the functional integral, thus generating a unique non-local theory expressed in terms of curvatures.

1. Introduction

One of the unconventional features of higher-spin gauge theories [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] is the peculiar role played by the corresponding generalised curvatures in the formulation of the dynamics.

Higher-spin linearised curvatures were introduced in [11, 12] for symmetric (spinor-) tensors, as proper extensions of the Maxwell field-strength and of the linearised Riemann curvature¹. They represent the *simplest* gauge invariant tensors that do not vanish on-shell, unless the field itself is pure gauge, with reference to an appropriate generalisation of the abelian gauge transformations of the photon and of the graviton.

The neat systematics underlying their construction, together with the relevance of their lower-spin counterparts, provide basic motivations for the idea that they might play some definite role in a theory of higher-spin gauge fields. On the other hand, due to the increasing number of derivatives needed in their definition, differently from the cases of spin 1 and spin 2, for spin $s \geq 3$ it is not possible to derive from them standard kinetic tensors. For this reason, it was long assumed that one should restrict to the non-geometric formulation of Fronsdal [13], involving only conventional, second-order, differential operators for the construction of free equations of motion for gauge fields of any spin (for an alternative approach see [14]). The price to pay with this choice is that, while no algebraic restrictions (other than symmetry properties of the gauge potential itself) are involved in the definition of higher-spin curvatures, the second-order formulation of [13] requires gauge fields $\varphi_{\mu_1 \dots \mu_s}$ constrained to be doubly traceless, and subject

¹ In the so-called “metric like” formalism, to be distinguished from the higher-spin generalisation of the Einstein-Cartan formulation of General Relativity, usually termed “frame-like approach” [1, 5].

to an abelian gauge transformation involving a rank- $(s - 1)$, traceless gauge parameter:

$$\delta \varphi_{\mu_1 \dots \mu_s} = \partial_{\mu_1} \Lambda_{\mu_2 \dots \mu_s} + \dots, \quad (1)$$

$$\begin{aligned} \varphi^{\alpha\beta}_{\alpha\beta\mu_5 \dots \mu_s} &\equiv 0, \\ \Lambda^\alpha_{\alpha\mu_3 \dots \mu_{s-1}} &\equiv 0. \end{aligned} \quad (2)$$

Under these conditions, it is indeed possible to show that the Fronsdal equation

$$\mathcal{F}_{\mu_1 \dots \mu_s} \equiv \square \varphi_{\mu_1 \dots \mu_s} - (\partial_{\mu_1} \partial^\alpha \varphi_{\alpha\mu_2 \dots \mu_s} + \dots) + (\partial_{\mu_1} \partial_{\mu_2} \varphi^\alpha_{\alpha\mu_3 \dots \mu_s} + \dots) = 0, \quad (3)$$

propagates the correct polarisations pertaining to a symmetric, massless, spin- s representation of the Poincaré group in D -dimensions². Generalisations of (3) to the case of tensors of mixed-symmetry were found by Labastida to also involve a proper set of algebraic conditions, on the traces of gauge fields and of their gauge parameters [16].

Whereas unusual, the Fronsdal constraints are instrumental in defining a Lagrangian theory in which the number of off-shell components is kept to a minimum. Moreover, the very existence [17] of consistent, non-linear generalisations of (3), makes it fair to say that there are no compelling reasons suggesting that one should try and get rid of them. On the other hand, simplifications might be expected in a framework where (2) are not to be assumed from the beginning, and in any case it would be rather unsatisfactory if the linearised geometry of [11, 12] were found not to admit deformations relevant for higher-spin non-abelian interactions³.

Indeed, after [12], the study of higher-spin geometry for symmetric tensors was further pursued in [21] and then extended to the case of mixed-symmetry tensor fields in [22]. Dynamical use of higher-spin curvatures was then proposed in [23, 24], where non-local Lagrangians and equations of motion for symmetric bosons and fermions were investigated, with no *a priori* reference to the Fronsdal formulation. The latter was then shown to be recovered performing the same partial gauge-fixing required in order to remove all non-localities. In a similar spirit, higher-derivative equations of motion for bosons of mixed-symmetry were formulated in [25, 26, 27] in terms of generalised field-strengths (antecedents of non-Lagrangian equations formulated via field-strengths can be found in [11, 14]) while proposals for corresponding higher-derivative or non-local actions were given in [26, 28]. Geometric equations of motion for mixed-symmetry fermions were discussed along similar lines in [4], while in the symmetric case, quadratic deformations of geometric Lagrangians were also constructed [29], providing direct generalisations of the Proca and Fierz-Pauli theories for the description of *massive* fields of any spin.

The basic indication obtained from these results is that, notwithstanding the presence of higher derivatives or non-localities, kinetic tensors built out of curvatures can still be used to describe freely propagating waves of any spin. On the other hand, allowing for non-local operators to be present leads to the consequence that gauge invariance alone is no more a sufficient criterion for the Lagrangian to be unique, and indeed in [30] infinitely many non-local kinetic tensors were shown to exist, for the case of symmetric bosons, besides the basic ones introduced in [23]. As a further selection rule allowing to distinguish among the various options, in [30] it was checked whether those theories reproduced the correct current exchanges between conserved sources, mediated by massless bosons of spin s . Under this requirement it was found

² To be precise, only tracelessness of the gauge parameter is needed in the counting of polarisations propagating in (3). Double-tracelessness of φ is then postulated to construct a suitable gauge-invariant Lagrangian. See also related discussions in [15, 12].

³ Clearly, linearised curvatures can be used to define abelian, Born-Infeld type vertices. They also appear naturally in the quantization of spinning particle models [18], as well as in the description of conformal higher spins [19] (for recent results and more references see [20]).

that, out of the infinitely many geometric Lagrangians consistent with gauge invariance, *only one* non-local theory actually possessed the correct propagator.

On the one hand, this result of uniqueness can be considered satisfactory, since it allows to define a true candidate linear limit for a theory of interacting higher-spins, possibly involving non-linearly deformed curvatures. However, it still leaves unanswered a few questions about the meaning of the whole procedure, given that the check of the propagator is only an *a posteriori* criterion of validity. In particular, the very fact that in the absence of sources infinitely many different equations appear to be consistent, calls for a better understanding of the rationale behind the non-localities of [23, 24, 30], in the spirit of clarifying the meaning of the manipulations involved.

With this purpose in mind, in this note we would like to suggest a different path for the definition of non-local Lagrangians expressed in terms of curvatures, and compare the outcome with the results of [23, 30]. The idea is very simple: we consider a framework where Fronsdal constraints are evaded by means of the introduction of auxiliary fields, so that the theory is still local, and its physical content can be determined using standard techniques. In particular, we resort to the Lagrangians proposed in [31], representing the simplest possible unconstrained ones for the case of symmetric (spinor-) tensors⁴. We then consider the gaussian functional integral for the corresponding theories, and perform the integration over the auxiliary fields. In this way we obtain “effective” non-local Lagrangians involving the physical field φ alone, whose unconstrained gauge invariance implies that they must be expressible in terms of higher-spin curvatures.

The advantage of this procedure is twofold: first, we define in this way an *a priori* criterion to select one member in the class of all possible non-local, geometric Lagrangians. In addition, starting from a theory whose spectrum is known by conventional analysis, we are able to unambiguously relate non-localities to the presence of non-physical fields in the initial local Lagrangians.

We investigate along these lines a few specific cases. After recalling basic facts about higher-spin curvatures and minimal local Lagrangians, which we do in Section 2, we study in Section 3 the form of the non-local effective action for symmetric bosons of spin 3 and spin 4 on flat space-time. We thus show that the integration over the auxiliary fields produces effective, non-local Lagrangians, coinciding with those selected in [30] as the only ones leading to the correct propagators. In addition, in Section 3.3 we broaden our analysis to include the case of (A)dS backgrounds, where we provide the first example of a non-local Lagrangian, for a spin-3 field. Finally, in Section 4 we move our attention to half-integer spins, studying the case of fermions of spin $\frac{5}{2}$. We find in this way the form of the non-local fermionic Lagrangian giving rise to the proper current exchange.

Similar ideas will be exploited in a forthcoming paper [35] with the purpose of analysing the geometrical content of higher-spin *triplets* ([24, 36, 9], and references therein). In the case of those systems, unconstrained gauge-invariance is related to the propagation of several irreducible representations⁵, so that, with certain qualifications, a direct correspondence with a sum of constrained Fronsdal Lagrangians can indeed be established [38]. Still, once the auxiliary fields are integrated away, the resulting actions must be expressible in geometrical terms. This suggests in particular that curvatures might play a role in the formulation of higher-spin theories, regardless of whether the Fronsdal-Labastida constraints are assumed or not.

⁴ Previous results leading to non-minimal unconstrained Lagrangians can be found in [32]. The generalisation of the local Lagrangians of [31] to the mixed-symmetry case is given in [33]. Constrained descriptions of mixed-symmetry massless fields in the frame-like approach on maximally symmetric backgrounds can be found in [34].

⁵ The unconstrained reduction of triplet Lagrangians to the case of irreducible spin s is discussed in [37]. In the frame-like approach, a discussion of the triplets and of their geometrical meaning can be found in [40].

2. Geometric Lagrangians and the issue of uniqueness

Here and in the next section we recall some basic facts, in order to fix the notation and to stress the conceptual issues at stake.

Following the construction of [12], for a rank-3 tensor subject to the abelian gauge transformation (1) the corresponding curvature is

$$\mathcal{R}_{\mu\mu\mu,\nu\nu\nu} = \partial_\mu^3 \varphi_{\nu\nu\nu} - \frac{1}{3} \partial_\mu^2 \partial_\nu \varphi_{\mu\nu\nu} + \frac{1}{3} \partial_\mu \partial_\nu^2 \varphi_{\mu\mu\nu} - \partial_\nu^3 \varphi_{\mu\mu\mu}, \quad (4)$$

in a notation where indices denoted with the same letter are to be understood as being completely symmetrised, without normalization factors, with the minimum numbers of terms required⁶, and where in particular $\partial_\rho^k = \underbrace{\partial_\rho \cdots \partial_\rho}_{k \text{ times}}$ is to be understood as the product of k gradients. For spin 4 the proper generalisation of (4) is

$$\mathcal{R}_{\mu\mu\mu\mu,\nu\nu\nu\nu} = \partial_\mu^4 \varphi_{\nu\nu\nu\nu} - \frac{1}{4} \partial_\mu^3 \partial_\nu \varphi_{\mu\nu\nu\nu} + \frac{1}{6} \partial_\mu^2 \partial_\nu^2 \varphi_{\mu\mu\nu\nu} - \frac{1}{4} \partial_\mu \partial_\nu^3 \varphi_{\mu\mu\mu\nu} + \partial_\nu^4 \varphi_{\mu\mu\mu\mu}, \quad (5)$$

while the general formula for spin s in the present notation looks

$$\mathcal{R}_{\mu_s, \nu_s} = \sum_{k=0}^s \frac{(-1)^k}{\binom{s}{k}} \partial_\mu^{s-k} \partial_\nu^k \varphi_{\mu_k, \nu_{s-k}}, \quad (6)$$

with obvious meaning for subscripts. As previously recalled, the basic property of the tensors (6) is their unconstrained gauge-invariance under the transformation

$$\delta \varphi_{\mu_1 \cdots \mu_s} = \partial_{\mu_1} \Lambda_{\mu_2 \cdots \mu_s} + \cdots, \quad (7)$$

that in our notation we would simply write as $\delta \varphi = \partial \Lambda$. In addition, they satisfy cyclic and Bianchi identities, reflecting the fact they define irreducible, two-row Young tableaux for $GL(D, \mathbb{R})$. All these properties remain valid if the field φ carries a spinor index as well, so that (6) are also suitable for the definition of a linearised fermionic geometry [12].

In [30] the curvature (4) was used to define a *one-parameter class* of candidate, non-local “Ricci tensors” for a spin-3 field⁷

$$\mathcal{A}_\varphi(a) = \frac{1}{\square} \partial \cdot \mathcal{R}' + a \frac{\partial^2}{\square^2} \partial \cdot \mathcal{R}'', \quad (8)$$

⁶ We use the “mostly-plus” space-time metric in d dimensions, denoted with η . Apart from the case of curvatures, whenever there is no risk of confusion all symmetrised indices are left implicit. Lorentz traces are denoted by “primes” or by numbers in square brackets, while divergences are denoted by “ $\partial \cdot$ ”. Combinatorial factors can be computed following the rules of [23, 24]. In the product of different tensors full symmetrisation of indices is always understood, with no weight factors. Useful combinatorial identities are

$$(\partial^p \varphi)' = \square \partial^{p-2} \varphi + 2 \partial^{p-1} \partial \cdot \varphi + \partial^p \varphi', \quad \partial^p \partial^q = \binom{p+q}{p} \partial^{p+q},$$

$$(\eta^k \varphi)' = [D + 2(s+k-1)] \eta^{k-1} \varphi + \eta^k \varphi', \quad \eta \eta^{n-1} = n \eta^n.$$

⁷ The subscript “ φ ” in \mathcal{A}_φ is used to distinguish the non-local, Ricci-like tensors from their local analogues, to be introduced in the next section. Those will be indicated with the symbol \mathcal{A} , without subscripts, and will depend on the field φ and on an auxiliary field α .

assumed to define basic equations of motion of the form

$$\mathcal{A}_\varphi(a) = 0. \quad (9)$$

A first investigation of the consistency of (9) led to the observation that, for almost any value of a , they can be shown to imply the “compensator” equations

$$\mathcal{F} - 3\partial^3\alpha_\varphi(a) = 0, \quad (10)$$

where $\alpha_\varphi(a)$ is a non-local tensor whose explicit form depends on a , transforming however always with the trace of the gauge parameter:

$$\delta\alpha_\varphi(a) = \Lambda'. \quad (11)$$

Thus, after a suitable gauge-fixing, infinitely many distinct non-local equations can be reduced to the Fronsdal form. In particular, the especially simple choice $a = 0$ reduces (9) to the equation

$$\frac{1}{\square}\partial \cdot \mathcal{R}' = 0, \quad (12)$$

a prototype for the results of the works [23, 24], where free higher-spin Lagrangians formulated in terms of curvatures were first proposed.

On the other hand, as shown in [30], for almost any value of a , the actions associated to the Ricci tensor $\mathcal{A}_\varphi(a)$ turn out to give the *wrong* propagator, thus suggesting that the correct counting of degrees of freedom might involve some further subtleties. Moreover, as recalled in the Introduction, *only one* Lagrangian was proven to provide the correct inverse kinetic operator, defined by the requirement that the current exchange between distant sources effectively projects them onto their transverse-traceless parts in $D - 2$ dimensions. For the spin-3 case the “correct” geometric theory is completely characterised by the following quantities:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\varphi\{\mathcal{A}_\varphi - \frac{1}{2}\eta\mathcal{A}'_\varphi\}, \\ \mathcal{A}_\varphi &= \frac{1}{\square}\partial \cdot \mathcal{R}' + \frac{\partial^2}{2\square^2}\partial \cdot \mathcal{R}'' = \mathcal{F} - 3\partial^3\alpha_\varphi, \\ \alpha_\varphi &= \frac{1}{3\square^2}\partial \cdot \mathcal{F}', \end{aligned} \quad (13)$$

where in particular second and third of (13) provide the expression of \mathcal{A}_φ in terms of curvatures as well as its compensator form. For spin 4 the proper Lagrangian and corresponding Ricci tensor are instead given by

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\varphi\{\mathcal{A}_\varphi - \frac{1}{2}\eta\mathcal{A}'_\varphi + \eta^2\mathcal{B}_\varphi\}, \\ \mathcal{A}_\varphi &= \frac{1}{\square}\mathcal{R}'' + \frac{1}{2}\frac{\partial^2}{\square^2}\mathcal{R}''' - 3\frac{\partial^4}{\square^3}\mathcal{R}^{[4]} = \mathcal{F} - 3\partial^3\alpha_\varphi, \\ \mathcal{B}_\varphi &= -\frac{3}{8}\frac{1}{\square}\mathcal{R}^{[4]} = \frac{1}{2}\{\frac{1}{\square}\partial \cdot \partial \cdot \mathcal{F}' - \mathcal{F}''\}, \\ \alpha_\varphi &= \frac{1}{3\square^2}\partial \cdot \mathcal{F}' - \frac{1}{3}\frac{\partial}{\square^3}\partial \cdot \partial \cdot \mathcal{F}' + \frac{1}{12}\frac{\partial}{\square^2}\mathcal{F}''. \end{aligned} \quad (14)$$

It is clear that the structure of the non-local operators involved in (13) and (14) must retain some special meaning, quite beyond the fact that they guarantee unconstrained gauge invariance of the corresponding Lagrangians, since, as we recalled, the latter can be achieved in several other ways. As a clue to uncover the rationale behind the solutions (13) and (14) we first recall the results of [31, 30], showing how it is possible to obtain full gauge invariance in the simplest way, within the framework of conventional local theories.

2.1. Minimal local theory

For a rank- s fully symmetric tensor the first step is to consider the unconstrained variation of the Fronsdal tensor \mathcal{F} ,

$$\delta \mathcal{F} = 3\partial^3 \Lambda', \quad (15)$$

and introduce a spin- $(s - 3)$ compensator field α , transforming as

$$\delta \alpha = \Lambda', \quad (16)$$

so that the local kinetic tensor

$$\mathcal{A} = \mathcal{F} - 3\partial^3 \alpha, \quad (17)$$

be identically gauge-invariant [24, 36]. Then, exploiting the Bianchi identity for \mathcal{A} ,

$$\partial \cdot \mathcal{A} - \frac{1}{2}\partial \mathcal{A}' = -\frac{3}{2}\partial^3 \{\varphi'' - 4\partial \cdot \alpha - \partial \alpha'\}, \quad (18)$$

it is not difficult to show that a gauge-invariant local Lagrangian can be written in the compact form [31, 30]

$$\mathcal{L} = \frac{1}{2}\varphi \{\mathcal{A} - \frac{1}{2}\eta \mathcal{A}'\} - \frac{3}{4}\binom{s}{3}\alpha \partial \cdot \mathcal{A}' + 3\binom{s}{4}\beta \{\varphi'' - 4\partial \cdot \alpha - \partial \alpha'\}, \quad (19)$$

where the Lagrange multiplier β transforms as $\delta\beta = \partial \cdot \partial \cdot \partial \cdot \Lambda$, while the tensor $\mathcal{C} \equiv \varphi'' - 4\partial \cdot \alpha - \partial \alpha'$ is the gauge-invariant completion of the double trace of φ . It is also manifest that Lagrangian (19) possesses the same physical content⁸ as Fronsdal's one.

3. Effective non-local Lagrangians for bosons

We would like to establish a direct link between (19) and the geometric Lagrangians of [30], recalled in the previous section. The basic observation is that the integration of the auxiliary fields α and β must define *non-local, effective Lagrangians* for the physical field φ , possessing the same physical content as (19). The unconstrained gauge invariance of the latter, on the other hand, implies that it should be possible to express the resulting effective Lagrangians in terms of curvatures.

Here we would like to perform explicitly this computation for the first few cases, to clarify the mechanism at work in some relatively simple examples. In particular we discuss spin 3 and spin 4 on flat space-time, and show that the corresponding non-local Lagrangians actually coincide with those providing the correct expression for the current exchange, eqs. (13) and (14). In addition, we apply our procedure to the study of a spin-3 field on (A)dS background, to then pass to the fermionic case, where we concentrate on the example of spin $\frac{5}{2}$ ⁹.

⁸ In particular, higher derivatives present in the kinetic operator of the compensator α are harmless. At any rate, it is possible to get rid of them slightly enlarging the field content of (19) [29, 33].

⁹ Since we are going to write functional integrals for gauge theories, we should in principle discuss the issue of gauge fixing, following for instance the Faddeev-Popov procedure. We shall not be concerned with this issue here, given that all we want to achieve is an effective Lagrangian for the field φ alone, whose proper quantization would require a separate discussion. For similar reasons, we will not discuss the Wick rotation of our theory to Euclidean space, where in principle functional integrals of gaussian theories can be given a rigorous definition (see e.g. [39]). Rather, we will limit ourselves to the formal use of the rules of gaussian integration in the Minkowskian region. This is tantamount to solving the equations for the auxiliary fields and substituting back in \mathcal{L} , with some attention to be paid on the field β , for which in the initial Lagrangian there is no full quadratic form.

3.1. Integrating out auxiliary fields: spin 3 on flat background

Let us denote with \mathcal{E}_φ the following combination of the Fronsdal tensor and its trace:

$$\mathcal{E}_\varphi = \mathcal{F} - \frac{1}{2}\eta \mathcal{F}'; \quad (20)$$

the functional integral for the unconstrained spin-3 theory, in the presence of an external source, can thus be written

$$\mathcal{Z}[\mathcal{J}] = \mathcal{N} \int \mathcal{D}\varphi \mathcal{D}\alpha e^{i \int d^d x \{ \frac{1}{2}\varphi \mathcal{E}_\varphi + \frac{9}{4}\alpha \square^2 \alpha - \frac{3}{2}\alpha \partial \cdot \mathcal{F}' - \varphi \cdot \mathcal{J} \}}, \quad (21)$$

where \mathcal{N} is an overall normalization. Performing the integration over α one obtains

$$\begin{aligned} \mathcal{Z}[\mathcal{J}] &= \mathcal{N}_\alpha \int \mathcal{D}\varphi e^{i \int d^d x \{ \mathcal{L}_{eff}(\varphi) - \varphi \cdot \mathcal{J} \}} \\ &= \mathcal{N}_\alpha \int \mathcal{D}\varphi e^{i \int d^d x \{ \frac{1}{2}\varphi \mathcal{E}_\varphi - \frac{1}{4}\partial \cdot \mathcal{F}' \frac{1}{\square^2} \partial \cdot \mathcal{F}' - \varphi \cdot \mathcal{J} \}}, \end{aligned} \quad (22)$$

where \mathcal{N}_α indicates a properly modified, field-independent, normalization factor, taking into account the gaussian integration over α . The corresponding non-local, effective Lagrangian is

$$\mathcal{L}_{eff}(\varphi) = \frac{1}{2}\varphi \mathcal{E}_\varphi - \frac{1}{4}\partial \cdot \mathcal{F}' \frac{1}{\square^2} \partial \cdot \mathcal{F}', \quad (23)$$

and can be shown to coincide, up to total derivatives, with the one defined in (13).

3.2. Integrating out auxiliary fields: spin 4 on flat background

For the unconstrained spin-4 case, rearranging terms in (19), we can write the functional integral as

$$\mathcal{Z}[\mathcal{J}] = \mathcal{N} \int \mathcal{D}\varphi \mathcal{D}\beta \mathcal{D}\alpha e^{i \int d^d x \{ \frac{1}{2}\varphi \mathcal{E}_\varphi + 3\beta \varphi'' + \frac{1}{2}\alpha A_\alpha \alpha + \alpha B_\alpha(\varphi, \beta) - \varphi \cdot \mathcal{J} \}}, \quad (24)$$

where

$$\begin{aligned} A_\alpha &= 18\square(\square + 3\partial \cdot \partial), \\ B_\alpha &= -6\{\partial \cdot \mathcal{F}' + \partial \partial \cdot \partial \cdot \varphi' - \square \partial \varphi'' - 4\partial \beta\}, \end{aligned} \quad (25)$$

are the relevant operators entering the gaussian integration over α .

Technically, the main issue consists in the inversion of the operator A_α , whose form depends crucially on the rank of the tensors on which it is supposed to act. In the present case what we need is A_α^{-1} when acting on the space of rank-1 tensors (the rank of the compensator α associated to the rank-4 field φ), whose explicit expression is

$$A_\alpha^{-1} = \frac{1}{18\square^2} \left(1 - \frac{3}{4} \frac{\partial}{\square} \partial \cdot \right). \quad (26)$$

Performing the integration over α in (24), and using (26), we thus obtain

$$\mathcal{Z}[\mathcal{J}] = \mathcal{N}_\alpha \int \mathcal{D}\varphi \mathcal{D}\beta e^{i \int d^d x \{ \frac{1}{2}\varphi \mathcal{E}_\varphi + 3\beta \varphi'' + B_\alpha(\varphi, \beta) \frac{1}{18\square^2} (1 - \frac{3}{4} \frac{\partial}{\square} \partial \cdot) B_\alpha(\varphi, \beta) - \varphi \cdot \mathcal{J} \}}. \quad (27)$$

In the latter expression we can reorganize the various terms, so as to make it explicit the quadratic form in the field β , allowing to perform the second gaussian integration:

$$\mathcal{Z}[\mathcal{J}] = \mathcal{N}_\alpha \int \mathcal{D}\varphi \mathcal{D}\beta e^{i \int d^d x \{ \frac{1}{2}\varphi \mathcal{E}_\varphi - C_\varphi + \beta \frac{1}{\square} \beta + \beta (3\varphi'' - \frac{1}{\square^2} \partial \cdot D_\varphi) - \varphi \cdot \mathcal{J} \}}, \quad (28)$$

where

$$\begin{aligned} C_\varphi &= D_\varphi \frac{1}{\square^2} D_\varphi + \frac{3}{4} \partial \cdot D_\varphi \frac{1}{\square^3} \partial \cdot D_\varphi, \\ D_\varphi &= \partial \cdot \mathcal{F}' + \partial \partial \cdot \partial \cdot \varphi' - \frac{1}{2} \square \partial \varphi''. \end{aligned} \quad (29)$$

Integrating over β we are formally left with a theory involving the field φ alone

$$\mathcal{Z}[\mathcal{J}] = \mathcal{N}_{\alpha, \beta} \int \mathcal{D}\varphi e^{i \int d^d x \{\mathcal{L}_{eff}(\varphi) - \varphi \cdot \mathcal{J}\}}, \quad (30)$$

where again the resulting non-local, effective Lagrangian

$$\mathcal{L}_{eff}(\varphi) = \frac{1}{2} \varphi \mathcal{E}_\varphi - D_\varphi \frac{1}{\square^2} D_\varphi - \frac{3}{4} \partial \cdot D_\varphi \frac{1}{\square^3} \partial \cdot D_\varphi - \frac{3}{2} \varphi'' \frac{1}{\square} \partial \cdot D_\varphi + \frac{9}{4} \varphi'' \square \varphi'', \quad (31)$$

can be shown to be equivalent to the corresponding expression in (14), for instance computing the equations of motion of (31) and verifying that they can be rearranged in the form

$$\mathcal{A}_\varphi - \frac{1}{2} \eta \mathcal{A}'_\varphi + \eta^2 \mathcal{B}_\varphi = 0, \quad (32)$$

with the various quantities in this expression defined in (14).

3.3. Integrating out auxiliary fields: spin 3 on (A)dS background

In [30] the minimal local Lagrangians (19), together with their fermionic counterparts, to be recalled in Section 4, were also generalised to the case of (A)dS backgrounds. On the other hand, the analysis of the non-local theory was considered only for the bosonic case on flat space-time. Here we would like to perform a first step towards a more general discussion, describing the non-local, unconstrained theory on (A)dS background, in the case of a spin-3 field. To this end, we follow two independent procedures.

First, we look for a non-local compensator α_φ , in the spirit of [30], and we make sure that the corresponding Lagrangian define the correct propagator. Then we consider the functional integral for the *local* theory, and we show that the integration of the compensator α produces indeed the same effective, non-local Lagrangian. In this fashion, we stress once again, the latter is given a clear interpretation, in terms of a theory whose dynamical content can be analysed using conventional methods.

3.3.1. Construction of the non-local compensator If we were to follow the very same procedure we went through in [30], the first step would be to resort to some proper, covariantised versions of the flat curvatures of [12] (see e.g. [41]). On the other hand, what we really need is a fully gauge invariant candidate ‘‘Ricci’’ tensor on (A)dS, that in the spin-3 case can be more simply constructed resorting to the gauge transformation of the covariantised Fronsdal tensor, when no assumptions are made on the traces of the gauge parameter¹⁰. To begin with, it might be convenient to recall the deformed (A)dS Fronsdal operator for a spin- s field [42]:

$$\mathcal{F}_L = \mathcal{F} - \frac{1}{L^2} \{[(3 - D - s)(2 - s) - s] \varphi + 2g \varphi'\}, \quad (33)$$

¹⁰In this way, looking for the kinetic tensor with the highest degree of singularity, we will be directly led to the solution possessing the correct propagator. If we were to start from curvatures, we would find proper (A)dS generalisations of the full family of Ricci-like tensors (8). This simplification, however, is special of the spin-3 case.

where D denotes the space-time dimension and

$$\mathcal{F} = \square \varphi - \nabla \nabla \cdot \varphi + \nabla^2 \varphi' \quad (34)$$

is the (A)dS-covariantized Fronsdal operator. The unconstrained variation of (33) under $\delta \varphi = \nabla \Lambda$, is

$$\delta \mathcal{F}_L = 3 \nabla^3 \Lambda' - \frac{4}{L^2} g \nabla \Lambda'. \quad (35)$$

Thus, considering the variation of $\nabla \cdot \mathcal{F}'_L$,

$$\delta \nabla \cdot \mathcal{F}'_L = 3 \square \{ \square - \frac{2}{L^2} (D + 1) \} \Lambda'. \quad (36)$$

it is relatively simple to identify a candidate non-local compensator

$$\alpha_{\varphi, L} = \frac{1}{3 \square \{ \square - \frac{2}{L^2} (D + 1) \}} \nabla \cdot \mathcal{F}'_L, \quad (37)$$

allowing to define a gauge invariant tensor in the form

$$\mathcal{A}_{\varphi, L} = \mathcal{F}_L - \{ 3 \nabla^3 - \frac{4}{L^2} g \nabla \} \alpha_{\varphi, L}. \quad (38)$$

The Bianchi identity satisfied by $\mathcal{A}_{\varphi, L}$,

$$\nabla \cdot \mathcal{A}_{\varphi, L} - \frac{1}{2} \nabla \mathcal{A}'_{\varphi, L} \equiv 0, \quad (39)$$

together with its further property

$$\nabla \cdot \mathcal{A}'_{\varphi, L} \equiv 0, \quad (40)$$

easily verified from

$$\mathcal{A}'_{\varphi, L} = \mathcal{F}'_L - \{ 3 \nabla \square - \frac{6}{L^2} (D + 1) \nabla \} \alpha_{\varphi, L}, \quad (41)$$

imply the existence of a non-local, gauge-invariant Lagrangian of the form

$$\mathcal{L} = \frac{1}{2} \varphi \{ \mathcal{A}_{\varphi, L} - \frac{1}{2} g \mathcal{A}'_{\varphi, L} \} - \varphi \cdot \mathcal{J}. \quad (42)$$

It is then straightforward to evaluate the propagator, once the field is coupled to a conserved source. Indeed, following the procedure described in [30, 43], from the Lagrangian equation

$$\mathcal{A}_{\varphi, L} - \frac{1}{2} g \mathcal{A}'_{\varphi, L} = \mathcal{J}, \quad (43)$$

we obtain

$$\mathcal{A}_{\varphi, L} = \mathcal{J} - \frac{1}{D} g \mathcal{J}'. \quad (44)$$

Then, introducing the Lichnerowicz operator \square_L defined by

$$\square_L \varphi = \square \varphi + \frac{1}{L^2} [s(D + s - 2) \varphi - 2g \varphi'], \quad (45)$$

we can rewrite $\mathcal{A}_{\varphi, L}$ for spin 3 as

$$\mathcal{A}_{\varphi, L} = (\square_L - \frac{4}{L^2}) \varphi + \nabla (\nabla \cdot \varphi - \frac{1}{2} \nabla \varphi') - (3 \nabla^3 - \frac{4}{L^2} g \nabla) \alpha_L. \quad (46)$$

From this expression we can compute the interaction between conserved currents, finding

$$\mathcal{J} \cdot \varphi = \mathcal{J} \frac{1}{\square_L - \frac{4}{L^2}} \mathcal{J}' - \frac{3}{D} \mathcal{J}' \frac{1}{\square_L - \frac{4}{L^2}} \mathcal{J}', \quad (47)$$

in agreement with the result found in [30] for the local counterpart of (42).

3.3.2. Integration of the compensator from the local theory While the computation of the propagator already represents a strong consistency check for the Lagrangian (42), we can further clarify its interpretation if we can show that it defines the effective Lagrangian of a more conventional local theory, once the compensator α is integrated away from the functional integral. To this end, following [30], let us write explicitly, for spin 3, the extension to (A)dS of the local unconstrained Lagrangian (19):

$$\mathcal{L} = \frac{e}{2} \varphi \{ \mathcal{A}_L - \frac{1}{2} g \mathcal{A}'_L \} - \frac{3e}{4} \alpha \nabla \cdot \mathcal{A}'_L, \quad (48)$$

where e denotes the determinant of the vielbein, g is the (A)dS metric, while the tensor \mathcal{A}_L has the same form as in (38), with α being in this case an independent Stueckelberg field, s.t. $\delta \alpha = \Lambda'$. In order to consistently eliminate the auxiliary field from the Lagrangian, we can start from

$$\mathcal{Z}[\mathcal{J}] = \mathcal{N} \int \mathcal{D}\varphi \mathcal{D}\alpha e^{i \int d^d x e^{\{ \frac{1}{2} \varphi \mathcal{E}_\varphi^L + \frac{9}{4} \alpha \square [\square - \frac{2}{L^2} (D+1)] \alpha - \frac{3}{2} \alpha \nabla \cdot \mathcal{F}'_L - \varphi \cdot \mathcal{J} \}}}, \quad (49)$$

where we defined

$$\mathcal{E}_\varphi^L = \mathcal{F}_L - \frac{1}{2} g \mathcal{F}'_L, \quad (50)$$

and perform the gaussian integration over α , obtaining the effective Lagrangian

$$\mathcal{L}_{eff}(\varphi) = \frac{1}{2} \varphi \mathcal{E}_\varphi^L - \frac{1}{4} \nabla \cdot \mathcal{F}'_L \frac{1}{\square [\square - \frac{2}{L^2} (D+1)]} \nabla \cdot \mathcal{F}'_L, \quad (51)$$

which, in its turn, can be shown to coincide with (42), up to total derivatives.

4. Effective non-local Lagrangians for fermions

Now we would like to extend our considerations to symmetric fermions. We start recalling the construction of the corresponding minimal local Lagrangians [31, 30], to then pass to the integration of the auxiliary field for the case of spin $\frac{5}{2}$, followed by the geometrical interpretation of the result.

4.1. Minimal local theory

The construction of local, unconstrained Lagrangians for symmetric spinor-tensors¹¹ ψ of rank s (and spin $s + \frac{1}{2}$) closely resembles the corresponding one for bosons, here sketched in Section 2.1. Under the transformation $\delta \psi = \partial \epsilon$ the unconstrained variation of the Fang-Fronsdal tensor [44],

$$\mathcal{S} = i (\partial \psi - \partial \psi), \quad (52)$$

is $\delta \mathcal{S} = -2i \partial^2 \psi$. Thus, we can build from \mathcal{S} the *fully* gauge invariant operator

$$\mathcal{W} \equiv S + 2i \partial^2 \xi, \quad (53)$$

where the rank- $(s-2)$ compensator ξ transforms as $\delta \xi = \psi$. The Bianchi identity for \mathcal{W} ,

$$\partial \cdot \mathcal{W} - \frac{1}{2} \partial \mathcal{W}' - \frac{1}{2} \partial \mathcal{W} = i \partial^2 \{ \psi' - 2 \partial \cdot \xi - \partial \xi' - \partial \xi \}, \quad (54)$$

¹¹ In the conventions here followed γ_0 is antihermitian, while the γ_i , $i = 1, 2, 3$ are hermitian. Moreover $\gamma_0 \gamma_\mu^\dagger \gamma_0 = \gamma_\mu$. An additional useful combinatorial rule is $\gamma \cdot (\gamma \psi) = (D + 2s) \psi - \gamma \psi$.

leads naturally to a second gauge-invariant spinor-tensor,

$$\mathcal{Z} \equiv i \{ \psi' - 2 \partial \cdot \xi - \partial \xi' - \not{\partial} \not{\xi} \}, \quad (55)$$

directly related to the triple γ -trace constraint on the fermionic gauge field ψ , absent in the Fang-Fronsdal formulation. The minimal flat-space Lagrangians of [31, 30] can then be recovered starting from the trial Lagrangians

$$\mathcal{L}_0 = \frac{1}{2} \bar{\psi} \{ \mathcal{W} - \frac{1}{2} \eta \mathcal{W}' - \frac{1}{2} \gamma \mathcal{W} \} + h.c., \quad (56)$$

and compensating the remainders in their gauge transformations with new terms involving the field ξ and the tensor \mathcal{Z} . The complete Lagrangian is finally

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \bar{\psi} \{ \mathcal{W} - \frac{1}{2} \gamma \mathcal{W} - \frac{1}{2} \eta \mathcal{W}' \} - \frac{3}{4} \binom{n}{3} \bar{\xi} \partial \cdot \mathcal{W}' \\ & + \frac{1}{2} \binom{n}{2} \bar{\xi} \partial \cdot \mathcal{W} + \frac{3}{2} \binom{n}{3} \bar{\lambda} \mathcal{Z} + h.c., \end{aligned} \quad (57)$$

where the Lagrange multiplier λ transforms according to $\delta\lambda = \partial \cdot \partial \cdot \epsilon$, in order for \mathcal{L} to be gauge invariant. Lagrangians for constrained spinor-tensors of *any* symmetry, together with their unconstrained extensions, were first presented in the second of [33].

4.2. Integrating out auxiliary fields: flat background

We would like to perform the integration over the auxiliary fields, and investigate the form of the corresponding effective non-local Lagrangian. For simplicity, we limit ourselves to the example of spin $s = \frac{5}{2}$, in which case (57) reduces to¹²

$$\mathcal{L} = i \bar{\psi} \mathcal{E}_\psi + 2i \bar{\xi} \square \not{\partial} \xi + i (\bar{\xi} \partial \cdot \not{\mathcal{S}} - \partial \cdot \not{\mathcal{S}} \xi), \quad (58)$$

where we defined

$$\mathcal{E}_\psi = \mathcal{S} - \frac{1}{2} \gamma \not{\mathcal{S}} - \frac{1}{2} \eta \mathcal{S}'. \quad (59)$$

Thus, considering the fermionic partition function in the presence of two grassmannian sources $\bar{\theta}$ and θ ,

$$\mathcal{Z} [\bar{\theta}, \theta] = \mathcal{N} \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}\bar{\xi} \mathcal{D}\xi e^{i \int d^d x \{ \mathcal{L} + \bar{\theta} \psi + \bar{\psi} \theta \}}, \quad (60)$$

and performing the fermionic gaussian integration over $\bar{\xi}$ and ξ we obtain the non-local effective Lagrangian

$$\mathcal{L}_{eff} (\bar{\psi}, \psi) = i \bar{\psi} \mathcal{E}_\psi + i \partial \cdot \not{\mathcal{S}} \frac{\not{\partial}}{2 \square^2} \partial \cdot \not{\mathcal{S}}, \quad (61)$$

which is equivalent, up to total derivatives, to the form

$$\mathcal{L}_{eff} (\bar{\psi}, \psi) = i \bar{\psi} \{ \mathcal{W}_\psi - \frac{1}{2} \gamma \mathcal{W}_\psi - \frac{1}{2} \eta \mathcal{W}'_\psi \}, \quad (62)$$

with the non-local, Dirac-like, kinetic tensor \mathcal{W}_ψ defined as

$$\mathcal{W}_\psi = \mathcal{S} - \partial^2 \frac{\not{\partial}}{\square^2} \partial \cdot \not{\mathcal{S}}, \quad (63)$$

and satisfying

$$\partial \cdot \mathcal{W}_\psi \equiv 0, \quad (64)$$

which is relevant to stress in view of the following discussion. Now we would like to make it explicit the content of (62) in terms of curvatures, and compare these results with those obtained in [23, 24, 29] on the geometry of fermionic theories.

¹²To avoid confusion with signs: the last term in (58) is to be interpreted as $\bar{S}_\mu \gamma^\mu$, and not as $\not{\mathcal{S}}^\dagger \gamma^0$.

4.3. Geometric interpretation and propagator

Non-local candidate Dirac-Rarita-Schwinger tensors were first proposed in [23, 24], in analogy with the corresponding bosonic quantities computed from the curvatures of [12]. The issue of fermionic geometry was then reconsidered in [29]; there it was shown that, even keeping to a minimum the degree of singularity of the non-local operators involved, in the fermionic case infinitely many independent tensors can actually be constructed.

In particular, for the case of spin $s = \frac{5}{2}$ of interest in this section, starting from the fermionic curvature

$$\mathcal{R}_{\mu\mu,\nu\nu} = \partial_\mu^2 \psi_{\nu\nu} - \frac{1}{2} \partial_\mu \partial_\nu \psi_{\mu\nu} + \partial_\nu^2 \psi_{\mu\mu}, \quad (65)$$

two independent, gauge-invariant, kinetic tensors can be constructed:

$$\begin{aligned} \frac{\not{\partial}}{\Box} \mathcal{R}'_2 &\equiv D_2, \\ \frac{1}{\Box} \partial \cdot \mathcal{R} &\equiv \hat{D}_2 \end{aligned} \quad (66)$$

whose expression in terms of the Fang-Fronsdal operator (52) is

$$\begin{aligned} i D_2 &= \mathcal{S} + \frac{\partial^2}{\Box} \mathcal{S}' - \frac{\partial}{\Box} \partial \cdot \mathcal{S}, \\ i \hat{D}_2 &= \mathcal{S} - \frac{1}{2} \frac{\partial}{\Box} \partial \cdot \mathcal{S}, \end{aligned} \quad (67)$$

while the corresponding candidate Dirac tensor proposed in [23, 24],

$$\mathcal{S}_2 = \mathcal{S} + \frac{1}{3} \frac{\partial^2}{\Box} \mathcal{S}' - \frac{2}{3} \frac{\partial}{\Box} \partial \cdot \mathcal{S}, \quad (68)$$

can be shown to be equivalent to the linear combination

$$\mathcal{S}_2 = \frac{1}{3} i D_2 + \frac{2}{3} i \hat{D}_2. \quad (69)$$

Moreover, in analogy with what already recalled for bosons in the previous sections, additional gauge invariant tensors can be constructed allowing for higher degrees of singularity in the corresponding non-local expressions. On the other hand, the analysis of the current exchange performed for bosons in [30] leads to expect that only one of these theories would display the correct propagator.

For spin $\frac{5}{2}$, in the local formulation of [31, 30], the field equation for ψ in the presence of a source takes the form

$$\mathcal{W} - \frac{1}{2} \gamma \mathcal{W} - \frac{1}{2} \eta \mathcal{W}' = \mathcal{J}, \quad (70)$$

where the gauge invariant spinor-tensor \mathcal{W} , defined in (53), satisfies the Bianchi identity (54) (where it is to be recalled that $\mathcal{Z} = 0$, due to the equation for $\bar{\lambda}$), implying in particular that, on-shell

$$\partial \cdot \mathcal{W} = 0, \quad (71)$$

thus accounting for the conservation of \mathcal{J} . Therefore, in order to reproduce the correct propagator, we should select a non-local Dirac tensor satisfying the same relations (54) and (71), as well as an equation of the same form as (70).

It is not difficult to check that the Lagrangian theories associated to Dirac-type kinetic tensors (67) (as well as any of their linear combinations) do *not* reproduce (70) nor (71). Differently,

the non-local theory obtained after integration of the compensators $\bar{\xi}$ and ξ , possesses exactly the correct properties, as manifest from (62) and (64). Moreover, while other, independent, non-local compensators could be constructed, such as

$$\hat{\xi}_\psi = -\frac{1}{2} \frac{1}{i \square} \mathcal{S}', \quad (72)$$

having the same gauge transformation as $\xi_\psi = -\frac{\partial}{2i\square^2} \partial \cdot \mathcal{S}$, nonetheless the corresponding kinetic tensor

$$\hat{\mathcal{W}}_\psi = \mathcal{S} - \frac{\partial^2}{\square} \mathcal{S}' \quad (73)$$

would *not* satisfy (54) and (71). This is another manifestation of the fact that, in the non-local case, it is actually possible to modify quantities by the addition of non-local gauge invariant tensors in such a way that, while gauge transformation properties are obviously preserved, the meaning of the whole construction gets modified in a crucial way. Indeed, there is *only one* geometric theory, for fermions of spin $\frac{5}{2}$, which propagates the correct number of degrees of freedom, and the procedure we followed in this paper can be seen as a safe way to derive it. Finally, its geometric interpretation is made manifest expressing the kinetic tensor (63) in terms of the curvature (65), according to

$$\mathcal{W}_\psi = \mathcal{S} - \partial^2 \frac{\partial}{\square^2} \partial \cdot \mathcal{S} = 2i \frac{1}{\square} \partial \cdot \mathcal{R} - i \frac{\partial}{\square} \mathcal{R}' + i \frac{\partial^2}{\square^2} \partial \mathcal{R}'' . \quad (74)$$

5. Summary and outlook

We computed effective, non-local Lagrangians for some specific examples of higher-spin gauge fields on flat and (A)dS backgrounds, performing the integration over non-physical fields in the corresponding local theories. The main goal of our computations has been to produce a *definition* of non-local, geometric Lagrangians, possibly devoid of ambiguities on their physical content, clarifying in particular the rationale behind the presence of inverse powers of the d'Alembertian operator in their kinetic tensors.

In this fashion, we were able to provide further support for the particular forms of geometric Lagrangians for symmetric bosons on flat backgrounds first given in [30]. Here we also proposed examples of geometric theories with correct propagators for fermions of spin $\frac{5}{2}$ on flat space-time, and for bosons of spin 3 on (A)dS backgrounds. These results point towards the possibility that a geometric description of interacting higher-spins, involving proper deformations of linearised curvature tensors, could emerge from a conventional, local theory, once all auxiliary fields (or at least a proper subset of them) are integrated away. In particular, in view of the geometrical interpretation of triplets to be given elsewhere [35], it might not be necessary to this end that the local theory be formulated in terms of *unconstrained* fields.

As for what concerns the latter, we leave for future work the generalisation of the results here presented to irreducible, unconstrained bosons and fermions of any spin (and symmetry), as well as the corresponding analysis of massive higher-spin fields.

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References

- [1] Vasiliev M A 2004 *Fortsch. Phys.* **52** 702 (*Preprint* hep-th/0401177)
- [2] Sorokin D 2005 *AIP Conf. Proc.* **767** 172 (*Preprint* hep-th/0405069)
- [3] Bouatta N, Compere G and Sagnotti A 2004 (*Preprint* hep-th/0409068)
- [4] Bandos I, Bekaert X, de Azcarraga J A, Sorokin D and Tsulaia M 2005 *JHEP* **0505** 031 (*Preprint* hep-th/0501113)
- [5] Sagnotti A, Sezgin E and Sundell P (*Preprint* hep-th/0501156)
- [6] Bekaert X, Cnockaert S, Iazeolla C and Vasiliev M A 2005 (*Preprint* hep-th/0503128)
- [7] Metsaev R R 2006 *Nucl. Phys. B* **759** 147 (*Preprint* hep-th/0512342)
- [8] Francia D and Sagnotti A 2006 *J. Phys. Conf. Ser.* **33** 57 (*Preprint* hep-th/0601199)
- [9] Fotopoulos A and Tsulaia M (*Preprint* 0805.1346 [hep-th])
- [10] Iazeolla C (*Preprint* 0807.0406 [hep-th])
- [11] Weinberg S 1965 *Phys. Rev.* **138** B988
- [12] de Wit B and Freedman D Z 1980 *Phys. Rev. D* **21** 358
- [13] Fronsdal C 1978 *Phys. Rev. D* **18** 3624
- [14] Siegel W and Zwiebach B 1987 *Nucl. Phys. B* **282** 125
- [15] Curtright T 1979 *Phys. Lett. B* **85** 219.
- [16] J. M. F. Labastida, *Nucl. Phys. B* **322** (1989) 185.
- [17] Vasiliev M A 1990 *Phys. Lett. B* **243** 378.
Vasiliev M A 2003 *Phys. Lett. B* **567** 139 (*Preprint* hep-th/0304049)
- [18] Howe P S, Penati S, Pernici M and Townsend P K 1988 *Phys. Lett. B* **215** 555.
- [19] Fradkin E S and Tseytlin A A 1985 *Phys. Rept.* **119** 233.
- [20] Bastianelli F, Corradini O and Latini E 2008 *JHEP* **0811** 054 (*Preprint* 0810.0188 [hep-th])
Marnelius R 2009 (*Preprint* 0906.2084 [hep-th])
Vasiliev M A 2009 (*Preprint* 0909.5226 [hep-th])
- [21] Damour T and Deser S 1987 *Annales Poincaré Phys. Theor.* **47** 277
- [22] Dubois-Violette M and Henneaux M 2002 *Commun. Math. Phys.* **226** 393 (*Preprint* math/0110088)
- [23] Francia D and Sagnotti A 2002 *Phys. Lett. B* **543** 303 (*Preprint* hep-th/0207002)
- [24] Francia D and Sagnotti A 2003 *Class. Quant. Grav.* **20** S473 (*Preprint* hep-th/0212185)
- [25] Bekaert X and Boulanger N 2002 *Commun. Math. Phys.* **245** 27 (*Preprint* hep-th/0208058)
- [26] de Medeiros P and Hull C 2003 *Commun. Math. Phys.* **235** 255 (*Preprint* hep-th/0208155)
- [27] Bekaert X and Boulanger N 2003 *Phys. Lett. B* **561** 183 (*Preprint* hep-th/0301243),
de Medeiros P and Hull C 2003 *JHEP* **0305** 019 (*Preprint* hep-th/0303036)
- [28] Bekaert X and Boulanger N 2007 *Commun. Math. Phys.* **271** 723 (*Preprint* hep-th/0606198)
- [29] Francia D 2008 *Nucl. Phys. B* **796** 77 (*Preprint* 0710.5378 [hep-th])
Francia D 2008 *Fortsch. Phys.* **56** 800 (*Preprint* 0804.2857 [hep-th])
- [30] Francia D, Mourad J and Sagnotti A 2007 *Nucl. Phys. B* **773** 203 (*Preprint* hep-th/0701163),
[31] Francia D and Sagnotti A 2005 *Phys. Lett. B* **624** 93 (*Preprint* hep-th/0507144)
- [32] Pashnev A and Tsulaia M 1998 *Mod. Phys. Lett. A* **13** 1853 (*Preprint* hep-th/9803207),
Buchbinder I L, Pashnev A and Tsulaia M 2001 *Phys. Lett. B* **523** 338 (*Preprint* hep-th/0109067)
- [33] Campoleoni A, Francia D, Mourad J and Sagnotti A 2009 *Nucl. Phys. B* **815** 289 (*Preprint* 0810.4350 [hep-th])
Campoleoni A, Francia D, Mourad J and Sagnotti A 2010 *Nucl. Phys. B* **828** 425 (*Preprint* 0904.4447 [hep-th])
Campoleoni A (*Preprint* 0910.3155 [hep-th])
- [34] Alkalaev K B, Shaynkman O V and Vasiliev M A 2004 *Nucl. Phys. B* **692** 363 (*Preprint* hep-th/0311164)
Skvortsov E D 2008 *JHEP* **0807** 004 (*Preprint* 0801.2268 [hep-th])
Boulanger N, Iazeolla C and Sundell P 2009 *JHEP* **0907** 013 (*Preprint* 0812.3615 [hep-th])
Zinoviev Yu M 2010 *Nucl. Phys. B* **826** 490 (*Preprint* 0907.2140 [hep-th])
- [35] Francia D (*to appear*)
- [36] Sagnotti A and Tsulaia M 2004 *Nucl. Phys. B* **682** 83 (*Preprint* hep-th/0311257)
- [37] Buchbinder I L, Galajinsky A V and Krykhtin V A 2007 *Nucl. Phys. B* **779** 155 (*Preprint* hep-th/0702161)
- [38] Fotopoulos A and Tsulaia M 2009 *JHEP* **0910** 050 (*Preprint* [arXiv:0907.4061 [hep-th]])
- [39] Glimm J and Jaffe A 1981 *Quantum Physics: A Functional Integral Point of View* (NY: Springer-Verlag)
- [40] Sorokin D P and Vasiliev M A 2009 *Nucl. Phys. B* **809** 110 (*Preprint* 0807.0206 [hep-th])
- [41] Manvelyan R and Rühl W 2008 *Nucl. Phys. B* **797** 371 (*Preprint* 0705.3528 [hep-th])
- [42] Fronsdal C 1979 *Phys. Rev. D* **20** 848.
- [43] Francia D, Mourad J and Sagnotti A 2008 *Nucl. Phys. B* **804** 383 (*Preprint* 0803.3832 [hep-th])
- [44] Fang J and Fronsdal C 1978 *Phys. Rev. D* **18** 3630